Sequentially Consistent Risk Measures

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There is a growing interest in risk measures that can be used to calculate solvency capital requirements for financial institutions in a dynamic framework. The present study discusses a mild notion of time-consistency between risk measurements of the same financial position at different time points in the future, called *sequential consistency* (Roorda & Schumacher, 2007). Such consistency is meaningful from both the investor's and the regulator's point of view, as it requires that current capital allocations reflect the acceptability or unacceptability of the position in future times. Formally, sequential consistency can be expressed as

$$\rho_t(X + \rho_{t+1}(X)) = 0$$

for a random variable $X \in \mathcal{L}^\infty$ and a conditional risk measure $\rho_t(\cdot)$.

Within the context of representation theorems for coherent and convex risk measures (e.g. Detlefsen & Scandolo (2005), Tutsch (2008)), we show that under mild conditions it is possible to build a class of risk measures that satisfy sequential consistency. Furthermore, working in a simple setting we present a sequentially consistent version of Tail-Value-at-Risk and provide numerical examples.

Reflecting regulatory practice, when calculating capital at time t, we substitute the final payoff X with its value at a future time point $t + \delta$, $E^Q[X \mid \mathcal{F}_{t+\delta}]$, where δ is a time-horizon of capital assessment. Thus, for a given valuation measure Q, a new risk measure is defined:

$$c_t^\delta(X) = \rho_t(E^Q[X \mid \mathcal{F}_{t+\delta}]). \tag{1}$$

Although (1) is generally not sequentially consistent anymore, different degrees of consistency can still be identified that improve on a naively used static risk measure.

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